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Demonstration of the Airy disk using photography and simple light sources

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A procedure for observing diffraction from a circular aperture using photography equipment and commonly available light sources is presented. Commonly called the Airy pattern, such a diffraction pattern is characterized by a central bright spot, with concentric bright and dark rings. The images obtained in this procedure are suitable for discussing and demonstrating Rayleigh’s criterion in introductory physics classes. It is shown that the patterns observed change as expected with wavelength and aperture. 

I. INTRODUCTION

Diffraction from a circular aperture is usually demonstrated with a coherent light source such as a laser and a small aperture. The interference pattern observed is described as a central bright spot, with concentric, alternating, bright and dark rings. In introductory physics classes, discussion of diffraction from a circular aperture is usually presented with a discussion of Rayleigh’s criterion, which simply states that two point sources will be just resolved if the central maximum of the first light source falls beyond the first minimum of the second light source. Typical homework problems in general physics usually deal with resolution of imaging equipment, including the human eye. Students often struggle making the conceptual connection between laser demonstrations and imaging-resolution problems. The other option is to simulate the effect. While simulations can be very helpful and intuitive, the procedure described herein demonstrates the far-field Fraunhofer diffraction phenomena directly using photography and simple light sources; it provides a real-world visual demonstration of diffraction and one can easily demonstrate resolution of light sources in the context of Rayleigh’s criterion. Further, this procedure readily demonstrates that real, diffraction limited optics do not provide crisp, well focused images of sufficiently small objects.

Following typical optics texts, the derivation of the expected light intensity considers plane waves of light incident on a circular aperture with radius \( a \). The light is allowed to be focused on a screen, some distance \( R \) from the aperture and making some angle \( \theta \) with the central optical axis. The intensity pattern observed is given by

\[
I(\theta) = I_0 \left( \frac{2J_1(x)}{x} \right)^2,
\]

where \( I_0 \) is the intensity of the central maximum of the pattern, \( J_1(x) \) is the Bessel function of the first kind, and \( x = ka \sin \theta \) with \( k = 2\pi/\lambda \) being the wave number. If the angle is small, \( \sin \theta \approx q/R \), where \( q \) is the radius of a dark fringe measured from the optical axis. For a light source at infinity, the observation distance equals the focal length of the lens used to focus the light through the aperture: \( R = f \). Thus,

\[
x = ka \sin \theta \approx \left( \frac{2\pi}{\lambda} \right) d \frac{q}{R} - \frac{\pi q}{\lambda} \frac{2a}{f} = \frac{\pi q}{2N},
\]

where \( N = f/2a \) is the relative aperture (the “f-number” or “f/#” in photography vernacular) of the lens. The Bessel function \( J_1(x) \) usually first appears in upper-level physics classes. However, beginning students can still work with it in spreadsheets such as Excel. If approached as a special function, like \( \cos x, \sin x, \) or \( e^x \), it is “just” a function that is called upon in the process of a calculation. Using a spreadsheet, students can make a theoretical curve for Eq. (1), as shown in Fig. 1.

The diffraction pattern resulting from a circular aperture has a bright region in the center, commonly called the Airy disk, and a series of concentric bright rings around the center, forming the Airy pattern. The intensity function given by Eq. (1) is zero when \( x = 3.8217, 7.0156, 10.1735, 13.3237, \) and so on. Thus, the central bright spot occurs between \(-3.8317 < x < 3.8317\), and there will be concentric dark rings at each value of \( x \) that makes the intensity zero.

The typical application of the Airy pattern in second semester general physics is the discussion of the Rayleigh criterion for resolving two point sources of light. The Rayleigh criterion effectively states that for two point sources of light to be just resolved, the first maximum of the first Airy pattern falls on top of the first minima of the second Airy pattern. In other words, the smallest angular separation that the two

![Fig. 1. Expected intensity for the Airy pattern as a function of the variable \( x = ka \sin \theta \), from Eq. (1).](http://dx.doi.org/10.1119/1.4927525)
light sources can have can be determined by considering the first zero of the Bessel function. Inserting $x = 3.8317$ into Eq. (2) yields the usual condition $\frac{1.22\lambda}{d} = \sin \theta$, where $d = 2a$ is the diameter of the circular aperture. Real example images for use in classroom discussion are difficult to find, with a common example being the resolution of two stars. Typical atmospheric turbulence, however, will prevent one from adequately photographing the Airy disk and higher order rings.

In the following discussion, a simple technique to observe the Airy pattern that uses simple photography equipment and commonly available light sources is presented. The technique allows students to observe the Airy disk and higher order rings up to the fourth order. This procedure presented allows students to directly see the effect of diffraction and diminished resolution in “every-day” images.

II. IMAGE ANALYSIS

The technique is as simple as taking a photograph of distant light sources through a zoom lens with a digital single-lens reflex (DSLR) camera at minimum aperture (maximum $N$). The equipment used in the following is a Canon 700D(T5i), a Canon EF 70–300 mm zoom lens, and a tripod. The scene photographed can be a typical nighttime cityscape. Given the wavelength sensitivity of the camera and the proliferation of high-pressure sodium lamps, mercury lamps, and modern LED lighting in traffic lights, the Airy pattern is easily observed. Care must be taken to ensure that the camera is properly focused, and we note that the camera’s auto focus may not be sufficient. Also, efforts should be taken to minimize vibration of the camera by using a heavy tripod. In the images below, the camera was manually focused using its “live-view” function. Further, to minimize vibrations, the mirror-lockup function is used. Finally, the camera is set to save the data in raw format to prevent the loss of information that occurs with JPG files.

An example image is presented in Fig. 2. The image was taken with focal length of $f = 300$ mm, a relative aperture of $N = 45$ (colloquially “f/45”), and exposed for 15 s. The camera raw file is opened directly with photo processing software, such as Photoshop. The example image is then cropped in two places as indicated by the squares and reproduced in Figs. 3 and 4.

Figure 3 shows a scene with a variety of high-pressure sodium and mercury lamps. The lamps in this scene are
approximately 3.5 km from the camera. Note that the Airy pattern is clearly visible around each lamp. Further, some lamps are easily resolved while others are too close together to be resolved, directly demonstrating the idea behind Rayleigh’s criterion.

Figure 4 shows a scene with modern traffic lights that use LED lighting, which has a less broad spectrum than older, incandescent lights. These lights are also about 3.5 km from the observer. Note that considering the size of the traffic lights, the average diameter of a patch of spatial coherence is likely slightly less than the camera aperture, but this issue can be remedied by looking at lights that are further away. Figure 4 also shows the processing used to analyze the patterns herein. Once opened in Photoshop, the image is split into the three component channels: red, green, and blue. Splitting an image is a well-defined procedure in Photoshop in which only the green information, for example, is saved to an image file. Typical DSLR cameras have peak sensitivity in the green to mimic the “acuity of the human visual system;” this is intentional and is accomplished by the standard Bayer sensor arrangement, which places two green sensitive detectors for every red and blue detector. Detailed sensitivity data for most cameras are available commercially, but to one significant figure the green information in an image will generally be twice as sensitive to light as that of red or blue. According to geometric quantum efficiency curves for similar Canon DSLR cameras, the green channel has a peak sensitivity at 520 nm with a 100-nm full-width-at-half-maximum (FWHM) value.

Once an image is split into its component images, each one is saved as a high quality JPG file. Unless stated otherwise, the green channel is used to analyze the Airy pattern. The open-source software program TRACKER is used to extract intensity as a function of pixel position, using the “line profile” tool. A sample “line” and its intensity profile are shown in Figs. 5 and 6, respectively. The line profile in Fig. 6 not only shows several minima as expected from Eq. (1) but also note that the central maximum is clipped at a value of 255 due to saturation of the detector (over exposure). Using the line profile from Fig. 6, local minima are identified and used to compute the radii of the dark rings. It is noted that the minima in intensity are not identically zero. The two primary reasons for this are that the incident light is not monochromatic and, to a lesser extent, vibrations in the camera effectively blur the information. Since the origin is arbitrary, the pixel coordinate for each pair of symmetric minima is subtracted and divided by two to obtain the radius. By analyzing multiple light sources, such as yellow sodium lamps in Fig. 3, several sets of measured radii are easily obtained. A sample of collected data are presented in Table I.

For comparison, the expected radii of the dark rings, \( q \), is computed using Eq. (2) with the knowledge of the pixel size and expected wavelength of light. The Canon 700D sensor has square pixels, with side length of 4.3 \( \mu \)m. High-pressure sodium lamps have a spectra differing from low-pressure sodium lamps used as wavelength standards. The most significant line in the high-pressure sodium lamp that falls within the sensitivity of the green sensor is \( \lambda = 570 \) nm. Using this information, the expected radii of each dark ring is also presented in Table I.

A. Wavelength

These data can be analyzed in a different, albeit somewhat unorthodox, manner. It is noted that the first five zeros of the Bessel function \((x = 3.8317, 7.0156, 10.1735, 13.3237, \text{and } 16.4706)\) can be approximated to better than 0.2% by the linear function

\[
x = 3.158m + 0.68,
\]

where \( m \) is the integer order number for the 1st, 2nd, etc., dark ring. Thus, setting
and re-arranging we obtain

\[ q_m = \frac{\lambda N}{\pi} \left( 3.158m + \frac{\lambda N}{\pi} 0.68 \right). \] (5)

Using Eq. (5), the measured dark fringe radius \( q_m \) is treated as the dependent variable and the order \( m \) is treated as the independent variable. Thus, the slope will yield the nominal wavelength \( \lambda \) responsible for the pattern. For non-monochromatic light sources, the nominal wavelength producing the pattern will depend on both the incident wavelength and the spectral sensitivity of the camera. This approach further assumes that the relative aperture \( N \) is absolutely known.

Figure 7 shows the measured radii from sodium and mercury lamps in Fig. 3 with an uncertainty of one pixel, along with linear regressions. The slopes yield an effective wavelength of \( \lambda = 540 \pm 20 \) nm for the sodium lamps and \( \lambda = 535 \pm 30 \) nm for the mercury lamps. It might be possible to fit the pixel data (e.g., Fig. 6) at local minima to reduce the uncertainty of the minima to less than one pixel; however, this may have a diminishing return on reducing the error of the slope in Fig. 7. It would appear that the camera’s sensitivity has a large bearing on the observed pattern rather than the wavelength of the incident light.

Attention is now turned to the analysis of the traffic lights in Fig. 4, which were also photographed when they were red (not shown). In a manner similar to that described above, at least three line profiles are extracted from the green channel when the lights were green, and three line profiles from the red channel when the lights were red. Unfortunately, the light-dark contrast in the red Airy patterns were not as distinct as the patterns in the green channel (this could be due to the relative sensitivity of these two channels); thus, only three radii are recorded for the red, with a larger error assigned to the radius of the third minima. Nonetheless, it is instructive to carry out the analysis described above. The data are shown in Fig. 8 with the slopes yielding \( \lambda = 450 \pm 20 \) nm and \( \lambda = 605 \pm 60 \) nm for the green and red lights, respectively. Thus, the Airy pattern radii changes as expected depending on the wavelength (although it is worth noting that the “green” traffic light is much closer to blue wavelengths and appears so in the color photograph of Fig. 4).

### B. Exposure

The effect of exposure on the resulting pattern is investigated using a different light source. For this experiment, a

![Fig. 7](image1.png)

![Fig. 8](image2.png)

### Table I. Measured radii (in pixels) of dark rings for sodium lamps from Fig. 3 (pixel size of 4.3 \( \mu m \)) and predicted radii assuming \( \lambda = 570 \) nm in Eq. (2) with \( N = 45 \).

<table>
<thead>
<tr>
<th>Order</th>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
<th>Average</th>
<th>Predicted</th>
<th>Percentage error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>7.5</td>
<td>7.5</td>
<td>7.7</td>
<td>7.28</td>
<td>5.4</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>13.5</td>
<td>13.5</td>
<td>13.3</td>
<td>13.32</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>19</td>
<td>19.5</td>
<td>18.5</td>
<td>18.8</td>
<td>19.32</td>
<td>2.5</td>
</tr>
<tr>
<td>4</td>
<td>24.5</td>
<td>…</td>
<td>25</td>
<td>24.8</td>
<td>25.30</td>
<td>2.2</td>
</tr>
</tbody>
</table>

\[ \lambda = 535 \pm 30 \text{ nm} \] for the mercury lamps. It might be possible to fit the pixel data (e.g., Fig. 6) at local minima to reduce the uncertainty of the minima to less than one pixel; however, this may have a diminishing return on reducing the error of the slope in Fig. 7. It would appear that the camera’s sensitivity has a large bearing on the observed pattern rather than the wavelength of the incident light.
small white LED flashlight is placed \( \approx 1.6 \) km away from the camera. The white LED has a broad spectral feature at 550 nm with a FWHM of approximately 75 nm. Convoluted with the spectral response of the green channel in the camera, this light is sufficiently monochromatic to produce an Airy pattern from the camera’s aperture. Including the illuminated reflector in the flashlight, the optical size of the object is about 1 cm. At this specified distance, the object has an angular size of about \( 6.25 \) \( \mu \)rad. Assuming that the lens (focal length \( f = 300 \) mm) is not diffraction limited, this object would produce an image size smaller than one pixel. However, the resulting image is quite similar to the previous images (see Figs. 4 and 5) and considerably larger than a single pixel.

To demonstrate the effect of exposure and to show that the pattern is consistent with that of a quasi-monochromatic source, line profiles are extracted from the green channel for three different exposures. These are overlaid and assigned an origin at the center of the main disk (see Fig. 9). Even at the shortest exposure, the central disk slightly over-exposes the detector, yet there is clearly a first and a second dark ring around the central maximum. The longest exposure saturated the detector such that the first dark ring is not observed, but higher order rings up to the fifth ring are apparent. Moreover, it appears that the minima for each exposure are consistent with each other. Using the minima for all three exposures, a wavelength analysis is performed as described above yielding an effective wavelength of 510 \( \pm 20 \) nm, again consistent with the peak response of the green sensors in the camera.

C. Aperture size

The effect of aperture size can also be analyzed. Using a distant sodium lamp (on a different occasion but in a manner similar to that presented in Fig. 3), two different photographs are acquired, one at \( f/45 \) and the other at \( f/32 \). Following the wavelength analysis above [see Eq. (5) and Fig. 7], the slope for the smaller aperture (larger \( N \)) is expected to be larger for the same light source, meaning that the radii for the \( m \)th ring will be larger. So, for the exact same light source, the slopes for the two different apertures are expected to be proportional to the ratio \( 45/32 \approx 1.406 \). Figure 10 shows the data collected (data from Table I are also included in the graph, as appropriate, because it was taken under the same conditions). The slopes shown in Fig. 10 give a ratio of 1.3 \( \pm 0.2 \), in reasonable agreement to what is expected.

D. Monochromatic light sources

Finally, to demonstrate that this procedure provides valid results, the procedure was repeated using known, monochromatic light sources. In this demonstration, three different
lasers were used: a HeNe laser ($\lambda = 633$ nm), a green laser pointer ($\lambda = 532$ nm), and a blue laser pointer ($\lambda = 405$ nm). The lasers were independently directed through a spatial filter with a 15-μm pinhole. While the output of the spatial filter also produces an Airy pattern of its own, the central maximum was directed at the same Canon 700D and lens as before. Thus, the pinhole appears to be a point light source to the camera and any interference effects observed will be due to the aperture(s) in the camera. To approximate a similar 6.25-μrad angular size as above, the camera is placed approximately 2.5 m away. Using the same technique as outlined above, the camera is used to photograph the pinhole with a variety of aperture settings with the three different lasers.

A representative, composite image of all three light sources taken at f/45 is shown in Fig. 11. The exposure times are different, given the different sensitivities to these different wavelengths. In these images, the exposure was determined empirically to just saturate the central maximum so that they may be visually compared. In a manner similar to the above, the images are cropped and split into separate components. In this exercise, the red channel is used for the 633-nm source, the green channel for the 532-nm source, and the blue channel for the 405-nm source. After the images are split into the desired channel, the radii of the discernible minima are determined and plotted against the order number. The data for the five trials using various light sources and aperture settings are shown in Fig. 12. Using the slopes of each set of data, the apparent wavelength is determined as before and presented in Table II. As may be gleaned from the results, this technique is certainly not an accurate method of determining wavelength but the results do scale appropriately with incident wavelength and provide a very visual demonstration of diffraction caused by a camera’s internal aperture.

### III. CONCLUSION

Demonstration of the Airy pattern is usually demonstrated through the use of monochromatic light sources and appropriately small apertures. The resulting pattern, projected on a screen, is easy to analyze. However, the connection between these experiments to everyday imaging problems such as photography is difficult for students to realize. Herein a simple procedure for demonstrating and observing the Airy pattern is presented using readily available photographic equipment. This procedure is possible even for quasi-monochromatic light sources because the camera’s internal sensors have a somewhat narrow spectral response. The light sources for the demonstration can be as simple as distant street lights or traffic lights, but it was also demonstrated using a white LED light. The effect is most easily seen using the green channel of the captured image because this information will be approximately twice as sensitive as that of the blue and red channels. However, this effect was also demonstrated to work in all three channels using judiciously selected monochromatic light sources. The patterns were shown to scale with wavelength and aperture size as expected.

To the extent that Eq. (3) is valid, the method of plotting radii against the order number is insensitive to the absolute assignment of the order number since it is the shape that is used to determine the wavelength. However, this method is not terribly accurate. First, for light radiating from other than monochromatic sources, the observed pattern will be a result of the incident light spectral distribution convoluted with the spectral sensitivity of the camera. It would appear for commonly available light sources that the camera’s wavelength sensitivity has a large impact on the apparent wavelength responsible for the observed patterns, particularly when working with the green channel. Second, the uncertainty in determining the radii of the minima is typically limited to one pixel. Fitting curves to obtain sub-pixel resolution will help, but so will analyze more line profiles as this will increase the number of data points, improving statistics. Another method that was not investigated is to use computer-vision algorithms—specifically, the method of the Hough transform—to fit circles to the minima in the image. This procedure would account for the entire pattern instead of just a few “slices.” Finally, since the lenses for commercial cameras are sealed, often with compound optics, the geometry of the optics cannot be verified to ensure that the underlying assumptions are correct. Similarly, the relative aperture $N$ cannot be independently verified without disassembling the lenses.

There are many possibilities to make improvements in this method and may make suitable exercises for advanced physics students. Inaccuracies aside, the greatest value of the method and the motivation for doing this experiment is the direct and simple visual presentation that can be afforded to the student. By capturing images of distant streetlights in a local setting, it is easy to demonstrate to students the effects of diffraction with real, recognizable images and discuss what is meant by Rayleigh’s criterion, thus providing an appropriate and memorable learning experience to students.

### Table II. Measured wavelengths using the appropriate slopes from Fig. 12.

<table>
<thead>
<tr>
<th>Actual source $\lambda$ (nm)</th>
<th>Aperture, $N$ (f/#)</th>
<th>Measured $\lambda$ (nm)</th>
<th>Percentage error</th>
</tr>
</thead>
<tbody>
<tr>
<td>405</td>
<td>45</td>
<td>394 ± 42</td>
<td>2.6</td>
</tr>
<tr>
<td>532</td>
<td>22</td>
<td>554 ± 61</td>
<td>4.2</td>
</tr>
<tr>
<td>532</td>
<td>45</td>
<td>528 ± 30</td>
<td>0.8</td>
</tr>
<tr>
<td>633</td>
<td>45</td>
<td>623 ± 12</td>
<td>1.6</td>
</tr>
<tr>
<td>633</td>
<td>40</td>
<td>658 ± 5</td>
<td>3.9</td>
</tr>
</tbody>
</table>
Kelvin Bridge

William Thomson, Lord Kelvin, developed this variant of the Wheatstone Bridge in 1862 to allow the measurement of low resistances. The circuit looks very much like that of the Wheatstone bridge, with the addition of an auxiliary pair of ratio arms. Outwardly, the bridge can always be recognized by the presence of a standard low resistance in the form of a long, thick rod of material, with a sliding contact; this can be seen running across the front of the instrument in the picture, which is 50 cm long. This instrument was made by James W. Queen of Philadelphia ca. 1890. (Notes and picture by Thomas B. Greenslade, Jr., Kenyon College)