

How does a light-mill work? (Original by Philip Gibbs July 1996)

In 1873, while investigating infrared radiation and the element thallium, the eminent Victorian experimenter Sir William Crookes developed a special kind of radiometer, an instrument for measuring radiant energy of heat and light. Crookes's Radiometer is today marketed as a conversation piece called a light-mill or solar engine. It consists of four vanes each of which is blackened on one side and silvered on the other. These are attached to the arms of a rotor which is balanced on a vertical support in such a way that it can turn with very little friction. The mechanism is encased inside a clear glass bulb which has been pumped out to a high, but not perfect, vacuum.

When sunlight falls on the light-mill the vanes turn with the black surfaces apparently being pushed away by the light. Crookes at first believed this demonstrated that light radiation pressure on the black vanes was turning it round just like water in a water mill. His paper reporting the device was refereed by James Clerk Maxwell who accepted the explanation Crookes gave. It seems that Maxwell was delighted to see a demonstration of the effect of radiation pressure as predicted by his theory of electromagnetism. But there is a problem with this explanation. Light falling on the black side should be absorbed, while light falling on the silver side of the vanes should be reflected. The net result is that there is twice as much radiation pressure on the metal side as on the black. In that case the mill is turning the wrong way.

When this was realised other explanations for the *radiometer effect* were sought and some of the ones that people came up with are still mistakenly quoted as the correct one. It was clear that the black side would absorb heat from infrared radiation more than the silver side. This would cause the rarefied gas to be heated on the black side. The obvious explanation in that case, is that the pressure of the gas on the darker side increases with its temperature creating a higher force on that side of the vane. This force would push the rotor round. Maxwell analysed this theory carefully presumably being wary about making a second mistake. He discovered that in fact the warmer gas would simply expand in such a way that there would be no net force from this effect, just a steady flow of heat across the vanes. So it is wrong, but even the Encyclopaedia Britannica gives this false explanation today. As a variation on this theme, it is sometimes said that the motion of the hot molecules on the black side of the vane provide the push. Again this is not correct and could only work if the mean free path between molecular collisions were as large as the container, but in fact it is typically less than a millimetre.

To understand why these common explanations are wrong think first of a simpler set-up in which a tube of gas is kept hot at one end and cool at the other. If the gas behaves according to the ideal gas laws with isotropic pressure, it will settle into a steady state with a temperature gradient along the tube. The pressure will be the same throughout otherwise net forces would disturb the gas. The density would vary inversely to temperature along the tube. There will be a flow of heat from the hot end to the other but the force on both ends will be the same because the pressure is equal. Any mechanism you might conjecture that would give a stronger force on the hot end than on the cool end with no tangential forces along the length of the tube cannot be correct since otherwise there would be a net force on the tube with no opposite reaction. The radiometer is a little more complex but the same principle should apply. No net force can be generated by normal forces on the faces of the vanes because pressure would quickly equalise to a steady state with just a flow of heat through the gas.

Another blind alley was the theory that the heat vaporised gases dissolved in the black coating which then leaked out. This outgassing would propel the vanes round. Actually, such an effect does exist but it is not the real explanation as can be demonstrated by cooling the radiometer. It is found that the rotor then turns the other way. Furthermore, if the gas is pumped out to make a much higher vacuum, the vanes stop turning. This suggests that the rarefied gas is involved in the effect. For similar reasons, the theory that the rotation is propelled by electrons dislodged by the photoelectric effect is also ruled out. One last incorrect explanation which is sometimes given is that the heating sets up convection currents with a horizontal component that turns the vanes. Sorry, wrong again. The effect cannot be explained this way.

The correct solution to the problem was provided qualitatively by Osborne Reynolds, better remembered for the "Reynolds number". Early in 1879 Reynolds submitted a paper to the Royal Society in which he considered what he called "thermal transpiration", and also discussed the theory of the radiometer. By "thermal transpiration" Reynolds meant the flow of gas through porous plates caused by a temperature difference on the two sides of the plates. If the gas is initially at the same pressure on the two sides, there is a flow of gas from the colder to the hotter side, resulting in a higher pressure on the hotter side if the plates cannot move. Equilibrium is reached when the ratio of pressures on either side is the square root of the ratio of absolute temperatures. This is a counterintuitive effect due to tangential forces between the gas molecules and the sides of the narrow pores in the plates. The effect of these thermomolecular forces is very similar to the thermomechanical effects of superfluid liquid helium. The liquid, which lacks all viscosity, will climb the sides of its container towards a warmer region. If a thin capillary is dipped into the superfluid it flows up the tube at such speed that a fountain effect is produced at the other end.

The vanes of a radiometer are not porous. To explain the radiometer, therefore, one must focus attention not on the faces of the vanes, but on their edges. The faster molecules from the warmer side strike the edges obliquely and impart a higher force than the colder molecules. Again these are the same thermomolecular forces that are responsible for thermal transpiration. The effect is also known as thermal creep since it causes gases to creep along a surface where there is a temperature gradient. The net movement of the vane due to the tangential forces around the edges is away from the warmer gas and towards the cooler gas with the gas passing round the edge in the opposite direction. The behaviour is just as if there were a greater force on the blackened side of the vane (which as Maxwell showed is not the case), but the explanation must be in terms of what happens not at the faces of the vanes but near their edges.

Maxwell refereed Reynolds's paper, and so became aware of Reynolds's suggestion. Maxwell at once made a detailed mathematical analysis of the problem, and submitted his paper, "On stresses in rarefied gases arising from inequalities of temperature", for publication in the Philosophical Transactions; it appeared in 1879, shortly before his death. The paper gave due credit to Reynolds's suggestion that the effect is at the edges of the vanes, but criticised Reynolds's mathematical treatment. Reynolds's paper had not yet appeared (it was published in 1881), and Reynolds was incensed by the fact that Maxwell's paper had not only appeared first, but had criticised his unpublished work! Reynolds wanted his protest to be published by the Royal Society, but after Maxwell's death this was thought to be inappropriate.

By the way. It *is* possible to measure radiation pressure using a more refined apparatus. To make it work you have to use a much better vacuum, suspend the vanes from fine fibers and coat the vanes with an inert glass to prevent out-gassing. When you succeed the vanes are deflected the other way as predicted by Maxwell. The experiment is very difficult but was first done successfully in 1901 by Pyotr Lebedev and also by Eeneest Nichols and Gordon Hull.

References

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